LIST OF CORE GRADUATE COURSES

1. Algebra

(1) Rings and Modules (Math 9023)
Rings of fractions and localization, Chinese Remainder Theorem, factorization in commutative rings, Euclidean algorithm, PIDs, algebraic integers, polynomials and formal power series, factorization in polynomial rings; Modules: generation, direct products and sums, freeness, presentations, modules over PIDs. Tensor algebras, exact sequences, projectivity, injectivity, Hom and duality, Zorn's Lemma, chain conditions.

Prerequisites: Introduction to Abstract Algebra (Math 3020), Group Theory (Math 3120), or equivalent.

(2) Representation Theory of Finite Groups (Math 9140)
Basic representation theory: irreducibility, complete reducibility, Schur's lemma, character theory, induced representations, etc. Possible further topics include: Fourier analysis on finite groups; applications to group theory, including theorems of Burnside; representations of the symmetric groups: partitions, Young tableaux, Young symmetrizers, Specht modules; probability and random walks on graphs.

Prerequisites: Group Theory (Math 3120), Intermediate Linear Algebra II (Math 2121), or equivalent.

(3) Commutative Algebra (Math 9141)
Prime and maximal ideals, the Zariski topology, localization, tensor products, flatness, integral dependence, going down and going up, chain conditions, Krull dimension, the Hilbert polynomial.

Prerequisite: Rings and Modules (Math 4123/9023), or equivalent.

(4) Homological Algebra (Math 9144)
Chain complexes and exact sequences; projective and injective resolutions; derived functors, Ext and Tor; filtrations and spectral sequences; examples for commutative and non-commutative algebra; examples which may include cohomology of groups, of Lie algebras, of sheaves; spectral sequences in topology.

Prerequisites: Rings and Modules (Math 4123/9023), or equivalent.

(5) Field Theory (Math 9020)
Automorphisms of fields, separable and normal extensions, splitting fields, fundamental theorem of Galois theory, primitive elements, Lagrange's theorem. Finite fields and their Galois groups, cyclotomic extensions and polynomials, applications of Galois theory to geometric constructions and solution of algebraic equations.

Prerequisite(s): Introduction to Abstract Algebra (Math 3020) and Group Theory (Math 3120) , or equivalent.

(6) Algebraic Geometry (Math 9053)
Affine and projective varieties, coordinate rings and function fields, birational correspondences, sheaves, dimension theory, regularity.

Prerequisites: Rings and Modules (Math 4123/9023), or equivalent.

2. Analysis

(1) Complex Analysis II (Math 9056)
Linear-fractional transformations, Schwarz's lemma, Reflection Principle, the Argument principle, the Riemann mapping theorem, Runge's theorem, the Mittag-Lefler and Weierstrass theorems.

Prerequisite: Complex Analysis I (Math 3124), or equivalent.

(2) Functional Analysis (Math 9054)
Hilbert spaces: L^2 spaces, orthogonal complements, dual spaces, Riesz representation theorem. Banach spaces: Hahn-Banach theorem, bounded linear operators, adjoints, closed graph and Banach Steinhaus theorems.

Prerequisites: Intermediate Linear Algebra I (Math 2120), Metric Space Topology (Math 3122),
Complex Analysis I (Math 3124), or equivalent.

(3) Introduction to Measure Theory (Math 9022)
Lebesgue measure, measurable sets and functions, Littlewood principles; the Lebesgue integral, basic convergence theorems, approximation theorems; measure spaces, signed measures, Radon-Nikodym Theorem.

Prerequisites: Metric Space Topology (Math 3122), or equivalent.

(4) Real Analysis (Math 9063)
Functions of several variables: continuity and differentiability, contraction mapping principle, the Inverse and Implicit Function theorems, higher order derivatives, integrals, differentiation of integrals depending on parameter. Distributions: partition of unity, test functions, generalized functions, basic properties, generalized functions as solutions of differential equations.

Prerequisites: Real Analysis II (Math 2123), Metric Space Topology (Math 3122), or equivalent.

3. Geometry and Topology

(1) Topology (Math 9021)
Topological spaces, neighbourhoods, bases, subspaces, product and quotient spaces, connectedness, compactness, separation axioms. Examples: surfaces, lens spaces, simplicial complexes. An introduction to covering spaces.

Prerequisites: Metric Space Topology (Math 3122), or equivalent.

(2) Algebraic Topology (Math 9052)
Homotopy, fundamental group, Van Kampen's theorem, covering spaces, simplicial and singular homology, homotopy invariance, long exact sequence of a pair, excision, Mayer-Vietoris sequence, degree, Euler characteristic, cell complexes, projective spaces. Applications include the fundamental theorem of algebra, the Brouwer fixed point theorem, division algebras, and invariance of domain.

Prerequisites: Group Theory (Math 3120) , General Topology (Math 3132), or equivalent.

(3) Calculus on Manifolds (Math 9055)
Manifolds (definition, examples, constructions), orientation, functions, partitions of unity, tangent bundle, cotangent bundle, vector fields, integral curves, differential forms, integration, manifolds with boundary, Stokes' theorem, submersions, immersions, embeddings, submanifolds, Sard's theorem, Whitney embedding theorem.

Prerequisites: Calculus 2503, Metric Space Topology (Math 3122), or equivalent.

(4) Differential Geometry (Math 9161)
Smooth manifold structure. tensors, Lie derivative, vector bundles, connections, curvature, principal bundles, de Rham cohomology, distributions, foliations, Frobenius theorem, Lie groups, group actions, classical matrix groups and their Lie algebras, low-dimensional exceptional isomorphisms, classical homogeneous spaces.

Prerequisites: Multivariable Calculus (Math 4155), Introduction to Abstract Algebra (Math 3020), or equivalent.

Note: All cross-listed courses have their undergraduate version as an anti-requisite.