Math Club At Western (MaCAW) 1st Annual Team Math Contest – Part 1
9 November 2018

- This contest consists of 10 questions worth 10 marks each, for a total of 100 marks. The questions in this contest are arranged in order of increasing difficulty.
- The time limit for this contest is **1 hour**.
- A **full solution** is required for each question. Marks are awarded for completeness and clarity.
- When applicable, express all answers as **simplified exact numbers**; for example, use $\pi + 2$ instead of 5.1415... , and use $1 - \sqrt{2}$ instead of –0.4142... .
- Complete **all** solutions on the lined paper provided. Write the name of **each** group member on the top of **every** page being submitted for grading. The lined paper provided may also be used for rough work. Do **not** include solutions and rough work on the same page.
- **No** materials are permitted during the contest other than writing materials.
- If you have any questions, raise your hand.
- If you finish the contest early, raise your hand. Your solutions will be collected from you.

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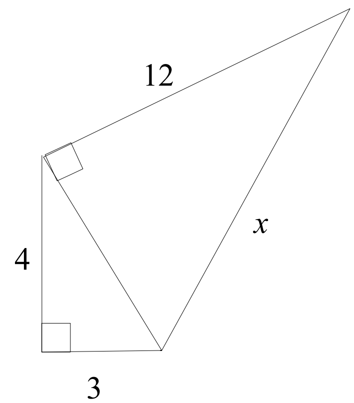
1. Determine the value of x.



2. Find all solutions of the equation $x + \frac{6}{x} = 5$.

3. Determine all complex numbers $z = x + iy$ such that $z^2 = i$.

4. Show that the sum of the first n positive integers is equal to $\frac{n(n+1)}{2}$.

5. Suppose that there is an infinite collection S of points lying inside a circle of radius 1. Show that for any $d > 0$, there exist points P_1 and P_2 in S such that the distance between P_1 and P_2 is less than d.

6. Prove or disprove: If $f : \mathbb{R} \to \mathbb{R}$ is a bounded, weakly decreasing function (i.e., $f(x) \leq f(y)$ whenever $x > y$), and f is differentiable on $\mathbb{R}$, then $\lim_{x\to\infty} f'(x) = 0$.

7. Show that the sum of the squares of the first n positive integers (that is, $\sum_{k=1}^{n} k^2$) is equal to $\frac{n(n+1)(2n+1)}{6}$.

8. Show that for any triangle, there exists a unique circle for which the 3 vertices of the triangle all lie on the circumference of the circle.

9. A quadratic function f with integer coefficients has two distinct roots, both of which are positive integers. The sum of the coefficients of f is a prime number, and for some positive integer n, $f(n) = -55$. Determine the two roots of f.

10. Find a 3-digit number n for which $n + 1$ is also a 3-digit number, and the 6-digit number formed by writing n followed by $n + 1$ is a perfect square. Show how the number was found.